Integration of some standard functions:

1. Integration of tanx.

$$I = \int \tan x = \int \frac{\sin x}{\cos x}$$

put $\cos x = t$

$-\sin x = \frac{dt}{dx}$

$\therefore dx = \frac{-dt}{\sin x}$

$$I = \int \frac{\sin x}{t} \cdot \frac{dt}{\sin x}$$

$$= -\int \frac{1}{t} = -\log t$$

$$= -\log \cos x$$

$$\therefore I = \log \sec x$$

2) Integration of cotx.

Soln: $I = \int \cot x$

Similarly $I = \log \sin x$

3) Integration of secx:

Soln $I = \int \sec x$

On multiplying numerator and denominator by $(\sec x + \tan x)$ we get

$$I = \int \frac{\sec x (\sec x + \tan x)}{(\sec x + \tan x)} dx$$

$$= \frac{\sec^2 x + \sec x \tan x}{(\sec x + \tan x)} dx$$

put $\sec x + \tan x = t$

On diffy $\sec x \tan x + \sec^2 x = \frac{dt}{dx}$

$$\therefore dx = \frac{dt}{(\sec x \tan x + \sec^2 x)}$$

$= \int \frac{\sec x(\sec x + \tan x)}{t} \cdot \frac{dt}{(\sec x \cdot \tan x + \sec^2 x)}$

$= \log t = \log(\sec x + \tan x)$ Ans

(5). Integration of $\operatorname{cosec} x$.

Let $I = \int \operatorname{cosec} x \, dx$

$= \int \frac{1}{\sin x} dx$

$= \int \frac{1}{2 \sin \frac{x}{2} \cdot \cos \frac{x}{2}} dx$

$= \frac{1}{2} \int \frac{1}{\left(\frac{\sin x/2}{\cos x/2}\right) \cos^2 x/2}$

$= \frac{1}{2} \int \frac{1}{\tan x/2 \cdot \cos^2 x/2}$

$= \frac{1}{2} \int \frac{\sec^2 x/2}{\tan x/2}$

Let $\tan x/2 = t$

$\frac{1}{2} \sec^2 x/2 \cdot \frac{2dt}{dx}$

$\therefore dx = \frac{dt}{\sec^2 x/2}$

$\therefore I = \frac{1}{2} \int \frac{\sec^2 x/2}{t} \cdot \frac{dt}{\sec^2 x/2}$

$= \frac{1}{2} \int \frac{1}{t} = \frac{1}{2} \log t$

$= \frac{1}{2} \log(\tan x/2)$

$= \log(\tan x/2)$ Ans

Q) Find $\int \frac{dx}{\sqrt{x^2+a^2}}$

Solution: Given

$$I = \int \frac{dx}{\sqrt{x^2+a^2}}$$

Let $x = a\tan\theta$ — (1)

$$1 = a\sec^2\theta \cdot \frac{d\theta}{dx}$$

$$\therefore dx = a\sec^2\theta\, d\theta$$

$$I = \int \frac{a\sec^2\theta}{\sqrt{a^2\tan^2\theta + a^2}}$$

$$= \int \frac{a\sec^2\theta}{a\sqrt{1+\tan^2\theta}}\, d\theta$$

$$= \int \frac{\sec^2\theta}{\sec\theta}$$

$$= \int \sec\theta$$

$$= \log(\sec\theta + \tan\theta) \quad —$$

Q.P $\sec^2\theta = \tan^2\theta + 1$

$$= \frac{x^2}{a^2} + 1 \qquad \text{(from eq 1)}$$

$$= \frac{x^2+a^2}{a^2} = \frac{\text{Hyp}^2}{\text{Base}^2}$$

$\therefore \sec\theta = \frac{\text{Hyp}}{\text{Base}} = \frac{\sqrt{x^2+a^2}}{a}$

Putting the values of $\sec\theta$ and $\tan\theta$ in the equation

$$= \log\left(\frac{\sqrt{x^2+a^2}}{a} + \frac{x}{a}\right)$$

Ans

Q) Integrate $\int \frac{\sin x}{\sqrt{1+\sin x}}\,dx$

Solution: $\int \frac{\sin x + 1 - 1}{\sqrt{1+\sin x}} = \int \frac{1+\sin x}{\sqrt{1+\sin x}} - \frac{1}{\sqrt{1+\sin x}}$

$= \int \sqrt{1+\sin x} - \frac{1}{\sqrt{1+\sin x}}$

$= \int \sqrt{\sin^2 x/2 + \cos^2 x/2 + 2\sin x/2 \cos x/2} - \frac{1}{\sqrt{\cos^2 x/2 + \sin^2 x/2 + 2\cos x/2 \sin x/2}}$

$= \int (\sin x/2 + \cos x/2) - \frac{1}{(\cos x/2 + \sin x/2)}$

$I_1 \qquad I_2$

Now consider I_1

$= \frac{1}{2}(\cos x/2 + \sin x/2)$

$= \frac{1}{2}(\sin x/2 - \cos x/2)$

Now, $I_2 = \frac{1}{(\sin x/2 + \cos x/2)}$

$= \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}(\sin x/2 + \cos x/2)}$

$= \frac{1}{\sqrt{2}} \cdot \frac{1}{\left(\frac{1}{\sqrt{2}}\sin x/2 + \frac{1}{\sqrt{2}} \cdot \cos x/2\right)}$

$\sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B$

$$= \frac{1}{\sqrt{2}} \frac{1}{\cos\frac{\pi}{4}\cdot\sin\frac{\pi}{2} + \cos\frac{\pi}{2}\cdot\sin\frac{\pi}{4}}$$

$$= \frac{1}{\sqrt{2}} \frac{1}{\sin\left(\frac{\pi}{4}+\frac{\pi}{2}\right)}$$

$$= \frac{1}{\sqrt{2}} \cdot \int \operatorname{cosec}\left(\frac{\pi}{4}+\frac{\pi}{2}\right)$$

$$= \frac{1}{\sqrt{2}} \cdot \log \frac{\tan\left(\frac{\pi}{4}+\frac{\pi}{2}\right)}{\frac{1}{2}}$$

$$= \frac{2}{\sqrt{2}} \log\left(\tan\left(\frac{\pi}{4}+\frac{\pi}{2}\right)\right)$$

$$= \sqrt{2} \log \tan\left(\frac{\pi}{4}+\frac{\pi}{2}\right)$$

$\therefore \quad I = I_1 + I_2$

$$= 2(\sin\pi/2 - \cos\pi/2) + \sqrt{2}\log\tan\left(\frac{\pi}{4}+\frac{\pi}{2}\right)$$

Ans

(Q) $\int \frac{1+\cos x}{\sin x}$

Solution: Given, $I = \int \frac{1+\cos x}{\sin x}$

$$= \int \frac{1}{\sin x} + \frac{\cos x}{\sin x}$$